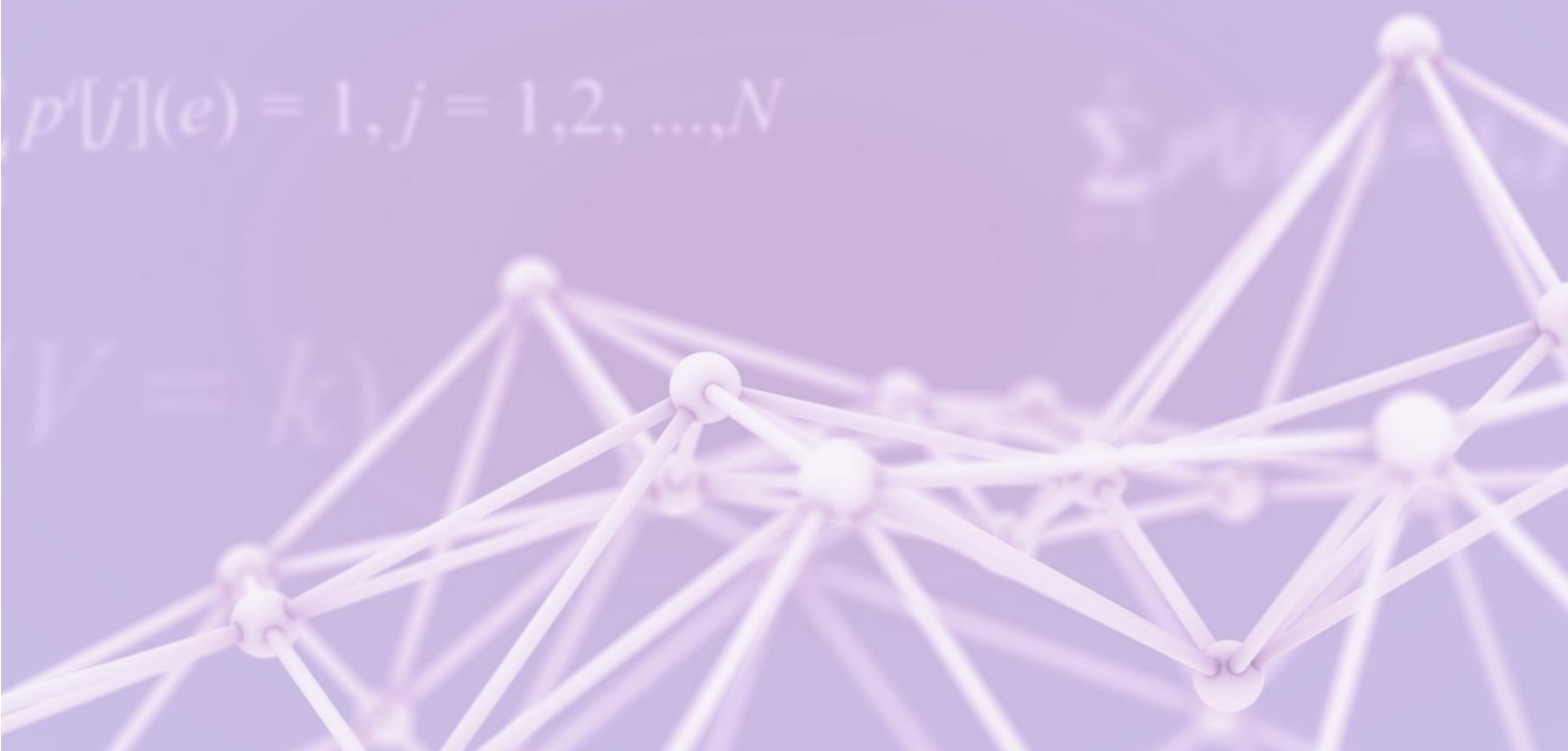# MATHEMATICAL DESCRIPTION OF CONSENSUS BUILDING IN THE DISTRIBUTED LEDGER TOKEN ACCOUNTING SYSTEM OF THE **BIT**BON SYSTEM

Source Data

$N$ — **number** of Assetboxes that participate in building consensus in the distributed ledger token accounting system of the **Bit**bon System,

$B(n)$ — **balance** of Assetbox number $n$, $n = 1,2,3,\ldots,N$,

$BB(n) = B(n) + B^{(1)}(n)$ — **base balance** where $B^{(1)}(n) = \sum_{i=1}^{[l^{[1]}(n)]} B(n(i))$ is the sum total of Assetbox balances of its first connection line, $B(n(i))$ is the balance of the $i$ Assetbox of the first line number $n(i)$.

**Analytical Expressions to Perform a Sequence of Actions Related to Calculating Remuneration for Activity as a Registrator**

1.

Let us calculate the (common value)

$B^{avg}$ — **average** of **balances** of the Assetboxes that participate in building consensus in the distributed ledger token accounting system of the **Bit**bon System,

$$B^{avg} = \sum_{n=1}^{N} B(n)/N.$$

2.

Let us calculate the (individual value) for each Assetbox number $n$

$b^r(n)$ — **relative balance** of the Assetbox,

$$b^r(n) = B(n)/B^{avg},$$

3.

Let us assign $z > 0$, an **influence coefficient of the first connection line** and function $f(b^r(n))$, an **influence regulator of the first line**, by selecting its parameters $a > d > 0$, $c > 0$.

It sets the value of entering the level of zero Assetboxes to zero.

It depends on the relative balance $b^r(n)$ of Assetbox number $n$.

It is continuous on the positive semiaxis and $f(0) = 0$, $f(\infty) = 1$:

$$f(x) = [x^\alpha + c * x^d]/(1 + x^\alpha), x \geq 0, a > d > 0.$$

4.

Based on the total balance $B^{(1)}(n)$ of the first connection line $B^{(1)}(n) = \sum_{i=1}^{[l^{[1]}(n)]} B(n(i))$ for each Assetbox number $n$, we calculate $R(n)$, the value of bringing Assetbox number $n$ to a certain level (adjusted base balance) determined by the value

$$R(n) = B(n) + z * B^{(1)}(n) * f(b^r(n)),$$

that depends on the balance $B(n)$ of Assetbox number $n$, as well as the total balance $B^{(1)}(n) = \sum_{i=1}^{[l^{[1]}(n)]} B(n(i))$ of its first connection line. This formula contains:

$z > 0$, an **influence coefficient of the first connection line** and function $f(b^r(n))$, an **influence regulator of the first line**, which depends on the relative balance of Assetbox number $n$.

5.

The issue of selecting the left boundary of the top (100th) level is solved using the generalized exponential distribution (Weibull distribution).

First, we need to express the parameter of the generalized exponential distribution through its median.

The generalized exponential distribution is as follows

$$F(x) = 1 - exp[-(\lambda x)^\alpha], x > 0.$$

Let us solve the equation $F(x) = 1/2$. Then, write down an equivalent equation

$$1/2 = 1 - exp[-(\lambda x)^\alpha].$$

Let us find

$$exp[-(\lambda x)^\alpha] = 1/2$$

and

$$\lambda x = (\ln 2)^{(1/\alpha)}.$$

Therefore

$$x^{(med)} = (\ln 2)^{(1/\alpha)}/\lambda.$$

The next step is to express the quantile for the probability 0.99 through the parameter of the exponential distribution.

Let us find the left boundary of level 100. We solve the equation $F(x) = 0.99$, or

$$0.99 = 1 - exp[-(\lambda x)^{\alpha}], \text{ or } exp[-(\lambda x)^{\alpha}] = 0.01.$$

Therefore

$$\lambda x = (\ln 100)^{(1/\alpha)}$$

and

$$x^{(0,99)} = (\ln 100)^{(1/\alpha)}/\lambda.$$

Then the relation is $x^{(med)} = (\ln 2)^{(1/\alpha)}/\lambda$, through the expression

$$\lambda = (\ln 2)^{(1/\alpha)}/x^{(med)}, \text{ we find}$$

$$x^{(0.99)} = (\ln 100)^{(1/\alpha)}/\lambda = x^{(med)}(\ln 100/\ln 2)^{(1/\alpha)} = x^{(med)} * [(\log 100)/(\log 2)]^{(1/\alpha)},$$

or

$$x^{(0.99)} = x^{(med)}[(\log 100)/(\log 2)]^{(1/\alpha)},$$

or

$$x^{(0.99)} = x^{(med)}[(\log 100)/(\log 2)]^{(1/\alpha)} = x^{(med)}(2/\log 2)^{(1/\alpha)}.$$

The left boundary of level 100 is expressed through the median of the exponential distribution as

$$x^{(left)}(100) = x^{(0.99)} = x^{(med)}(2/\log 2)^{(1/\alpha)}.$$

The next step is to determine the exponent of the distribution.

In order to determine the exponent of the generalized exponential distribution, we use the expression for the expected value of a random variable with this distribution:

$$F(x) = 1 - exp[-(\lambda x)^{\alpha}], x > 0.$$

It equals $M(1) = (1/\lambda)\Gamma(1 + 1/\alpha)$.

The expression for the median is

$$x^{(med)} = (\ln 2)^{(1/\alpha)}/\lambda,$$

we find

4

$$M(1)/x^{(med)} = \Gamma(1 + 1/\alpha)/(\ln 2)^{(1/\alpha)}.$$

Let us calculate

$$M(1)/x^{(med)} = A.$$

Let us solve the equation

$$\Gamma(1 + 1/\alpha)/(\ln 2)^{(1/\alpha)} = A$$

of the relatively known exponent $\alpha$. Here

$$\Gamma(1 + 1/\alpha) = \int\limits_{t>0} t^{(1/\alpha)} \exp(-t)dt$$

is the value of gamma function in the point $1 + 1/\alpha$.

Let us create the process of finding the expression

$$(\ln 2)^{(1/\alpha)} \int\limits_{t>0} t^{(1/\alpha)} \exp(-t)dt$$

with the previously assigned accuracy, the equation

$$\Gamma(1 + 1/\alpha)/(\ln 2)^{(1/\alpha)} = A$$

is solved using bisection or the method of golden section.

Let us use $\alpha = 1/3$ for the initial structure.

As an example.

If $x^{(med)} = 30$, $\alpha = 1/3$

$$x^{(left)}(100) = 30 * (2/\log 2)^3 = 30 * (6.64385618977472469574\dots)^3 = 8797.9588\dots$$

6.

Let us assign

$\Delta = x^{left}(1)$ is the left boundary of the first level, the value the crediting of remuneration starts from,

$A(1)$ — length of the first level interval,

$L$ — number of levels (in this case $L = 100$),

$D = x^{left}(L)$ — left boundary of the top level.

Let us determine:

$$S = [(D - \Delta)/A(1)]^{[1/(L-2)]}$$

the **range extender** of level intervals (starting from the second one).

For all levels, starting from level 2 ($2 \leq k \leq L$), the left interval boundary of level $k$ equals:

$$x^{left}(k) = x^{left}(1) + A(1) * S^{(k-2)} = \Delta + A(1) * S^{(k-2)}, 2 \leq k \leq L.$$

To illustrate this

Let us calculate the length $\Delta(k)$ of the level interval $k$, $1 \leq k \leq L$.

If $k = 1$, the length of the first level interval equals $A(1)$:

$$\Delta(1) = x^{left}(2) - x^{left}(1) = [x^{left}(1) + A(1) * S^{(2-2)}] - x^{left}(1) = A(1).$$

If $k = 2$, the length of the second level interval equals:

$$\Delta(2) = x^{left}(3) - x^{left}(2) = A(1) * S^{(3-2)} - A(1) = A(1) * (S - 1).$$

If $k = 3$, the length of the third level interval equals:

$$\Delta(3) = x^{left}(4) - x^{left}(3) = A(1) * S^{(4-2)} - A(1) * S^{(3-2)} = A(1) * (S^2 - S).$$

If $k = 4$, the length of the fourth level interval equals:

$$\Delta(4) = x^{left}(5) - x^{left}(4) = A(1) * S^{(5-2)} - A(1) * S^{(4-2)} = A(1) * (S^3 - S^2).$$

If $k = L - 1$, the length of the $L - 1$st level interval equals:

$$\Delta(L - 1) = x^{left}(L) - x^{left}(L - 1) = A(1) * S^{(L-2)} - A(1) * S^{(L-3)} = A(1) *$$
$$(S^{(L-2)} - S^{(L-3)}).$$

The left boundary of the $L$ level interval equals:

$$x^{left}(L) = x^{left}(1) + A(1) * S^{(L-2)} = \Delta + A(1) * S^{(L-2)} = \Delta + A(1)[(D - \Delta)/A(1)] = D.$$

The relation of lengths of adjacent intervals:

$$\Delta(k + 1)/\Delta(k), 2 \leq k \leq L - 2,$$

starting from the second and ending with $L - 2$nd equals $S$:

$$\Delta(k + 1)/\Delta(k) = [S^{(k)} - S^{(k-1)}]/[S^{(k-1)} - S^{(k-2)}] = S$$

for all $2 \leq k \leq L - 2$.

7.

We calculate **level percentage coefficients** $r(k)$ (**level percentages** $100 * r(k)$) for each level $k$,$k = 1, \ldots, L$:

$$r(k) = 0.3 + 0.7 * [(k-1)/(L-1)]^\beta,$$

$k = 1, \ldots, L$, while the power parameter $\beta > 0$ determines the growth speed near small and big level values (downward or upward convexity of the level coefficients chart). If $\beta = 1$, the growth speed is constant (points on the chart are in a straight line).

8.

Let us calculate the **base power** $W^b(n)$ of Assetbox $n$ for each Assetbox number $n$ using the formula:

$$W^b(n) = max[0.25 * B(n); W],$$

where

$$W = min[B(n); 0.25 * X]$$

is a minimum of the balance $B(n)$ of the Assetbox number $n$ and the sum of minimums $min[B(n); B^1(n(i))]$ of its balance $B(n)$ and balances $B^1(n(i)) = B(n(i))$ of all the nodes of the first line of connection to Assetbox $n$ multiplied by the coefficient 0.25:

$$X = \sum_{i=1}^{[l^{[1]}(n)]} min[B(n); B^1(n(i))].$$

In order to do that, let us perform the following sequence of actions.

8.1. Using the superscripts $1, \ldots$, we assign numbers to the levels (lines) of Assetboxes connected to the researched Assetbox.

8.2. We calculate the secondary variable:

$$X = \sum_{i=1}^{l} min[B(n); B^1(n(i))],$$

where $l$ — number of Assetboxes in the first line of the Assetbox;

$B^1(n(i))$ — balance of the $i$ Assetbox of the first line number $n(i)$.

8.3. We calculate the secondary variable:

$$W = min[B(n); 0.25 * X].$$

8.4. We determine the base power:

$$W^b(n) = max[0.25 * B(n); W].$$

Using this way of calculating base power, on condition that the first line of the researched Assetbox contains four Assetboxes with the balances equal to that of the researched one, this Assetbox will receive the maximum possible base power. There are other possible ways of structuring that would give the Assetbox the maximum possible base power for its balance.

9.

Let us enter the **adjusted social power** $W^{ns}(n)$ for each Assetbox number $n$, which is determined by the source contents of the node $n$, Assetboxes numbers $n(1), n(2), \ldots, n(l^{[1]})$ of the first line of connection $L^{[1]}(n)$ to the node $n$ (they provide the base power for entering a level) and contents of the Assetboxes of all its connection branches below the first line connected to the Assetbox with the level percentage lower than that of the Assetbox number $n$ (common value $\Pi(n)$).

$$W^{ns}(n) = \sum_{i=(l^{[1]}+1)}^{(Z(n))}[r(n) - C^*(n(i), n^{[1]}(\pi), n)] * B(n(i)) * I[r(n) > C^*(n(i), n^{[1]}(\pi), n)],$$

where $Z(n) = l^{[1]} + \Pi(n)$. Here, the variable $C^*(n(i), n^{[1]}(\pi), n)$ is determined as follows. There is a singular connection path from the node $n(i)$, located below the first line of connection to the Assetbox $n$, to the node $n$:

$$\pi(n^{[s]}(i), n)$$

that equals:

$$[n^{[s]}(i) \to n^{[s-1]}(\pi(n^{[s]}(i), n)) \to \ldots \to n^{[2]}(\pi(n^{[s]}(i), n)) \to n^{[1]}(\pi(n^{[s]}(i), n)) \to$$
$$n^{[0]}(\pi(n^{[s]}(i), n))],$$

or simplified as:

$$[n^{[s]}(i) \to n^{[s-1]} \to \ldots \to n^{[2]} \to n^{[1]} \to n^{[0]}]$$

that starts at the node $n(i) = n^{[s]}(i)$ at the bottom level $[s]$ and ends at the node $n = n^{[0]}(\pi(n^{[s]}(i), n)) = n^{[0]}$ at the top one, at level 0 from its point of view.

While

$$C^*(n(i), n^{[1]}(\pi), n) = max[r(n^{[1]})r(n^{[2]}), r(n^{[3]}), \ldots, r(n^{[s-1]}), r(n^{[s]})]$$

is the maximum of all level percentage coefficients

$$r(n^{[1]}), r(n^{[2]}), r(n^{[3]}), \ldots, r(n^{[s-1]}), r(n^{[s]})$$

of the nodes

$$n^{[1]}, n^{[2]}, n^{[3]}, \ldots, n^{[s-1]}, n^{[s]}$$

on the path

$$\pi(n^{[s]}(i), n) = [n^{[s]} \to n^{[s-1]} \to \ldots \to n^{[2]} \to n^{[1]} \to n^{[0]}]$$

of connections from the node $n(i) = n^{[s]}(i) = n^{[s]}$ to the node $n = n^{[0]}(\pi(n^{[s]}(i), n)) = n^{[0]}$.

The sum total is calculated using all the branches below the first line of connection to Assetbox number $n$, the level percentage of which is lower than that of Assetbox number $n$. No branches of the Assetbox with the level percentage lower than that of Assetbox number $n$ are used in calculations.

10.

Let us calculate the **majorizing social power** $W^{ms}(n)$ for Assetbox number $n$:

$$W^{ms}(n) = \sum_{i=(l^{[1]}+1)}^{(Z(n))} [r^{max} - C^*(n(i), n^{[1]}(\pi), n)] * B(n(i)) * I[r^{max} > C^*(n(i), n^{[1]}(\pi), n)].$$

The majorizing social power of node $n$ is different from the adjusted social power due to the use of the variable $r^{max} = 1$ instead of the variable $r(n)$.

11.

Then we calculate the social power $W^s(n)$ of the node $n$ based on the adjusted social power $W^{ns}(n)$ using the formula:

$$W^s(n) = W^{ns}(n) * g(C^o(n)).$$

Here we introduce the **social normalizing function** $g(x)$ to ensure the **social direction** of providing, which equals:

$$g(x) = [h + (1 - h)(1 + px^\eta)q^{(1-x)}],$$

$$0 < h < 1, q > 1, p \geq 0, \eta \geq 0.$$

Here $h$ — level of unachievable minimum of the social normalizing function;

$p$ — regulator of the wavelet of the social normalizing function chart;

$\eta$ — indicator of the power that ensures the wavelet of the function chart;

$q$ — base of an exponential social normalizing function, which ensures its movement towards an unachievable minimum $h$.

The **argument** $x$ of the social normalizing function is represented by the variable:

$$C^o(n) = W^{ms}(n)/B^{avg},$$

which equals the relation between the **majorizing social power** $W^{ms}(n)$ and the average $B^{avg}$ of the Assetbox balances.

12.

Let us calculate the **power** $W(n)$ for each Assetbox number $n$ as a sum of base and social powers of the node $n$:

$$W(n) = W^b(n) + W^s(n)$$

13.

The received sum $S$ is split among all Assetboxes

$$S = \sum_{n=1}^{N} S(n)$$

in direct proportion to their powers $W(n)$ using the formula for calculating **remuneration** $S(n)$ *for the* $n$-th Assetbox:

$$S(n) = S * W(n)/W,$$

where

$$W = \sum_{n=1}^{N} W(n)$$

is the sum of powers of all Assetboxes.

The values of parameters in the realized model.

$$z = 1$$
$$a = 1$$
$$\alpha = 1/3$$
$$c = 0.8$$
$$d = 1/3$$
$$r^{min} = 0.3, L = 100, \beta = 0.5$$
$$q = 2, p = 1.5, \eta = 1.46, \Delta = 10^{-3}, A(1) = 1, h = 0.05$$

# Parameters of the Formulas for Calculating Remuneration for Activity as a Registrator

$\Delta = 10^{-3}$ — minimum Assetbox balance required to receive remuneration.

$I(1) = 1$ — length of the first level interval.

$k = 1$ — first line value coefficient.

$a = 1$ — power of the main summands in an endless asymptotic.

$c = 0.8$ — coefficient that regulates the value of the function in unity.

$d = 1/3$ — exponent that allows setting the value of entering the level of zero Assetboxes to zero.

$r^{min} = 0.3$ — minimum value of level percentage coefficients.

$L = 100$ — tables of rank percentages of Assetbox powers and correspondence of the Assetbox power and the first line to a certain rank are calculated in a dynamic manner based on the maximum power in the system. As the number of levels changes, the boundaries of level intervals also change.

$\beta = 0.5$ — power parameter that determines the level of growth near small and big level values; must be higher than 0.

$h = 0.05$ — level of an unachievable minimum (bottom boundary) of a social function; must be higher than 0, but lower than 1.

$p = 1.5$ — regulator of the wavelet of the normalizing function.

$r = 1.46$ — power for the argument of the social function.

$q = 2$ — base of an exponential social normalizing function $q$, which ensures its movement towards an unachievable minimum $h$.