Online Appendix for Debortoli, Kim, Lindén and Nunes "Designing a Simple Loss Function for Central Banks: Does a Dual Mandate Make Sense?"

Appendix A The Linear Quadratic Approximation

In this appendix, we provide the details of the linear quadratic approximation that was used in our paper. We show that our algorithm can handle the case of a distorted steady state and generates the correct linear approximation. In addition, we provide conditions under which our legitimate linear quadratic approximation approach and a simpler illegitimate approach provide the same results.

A.1 A General Non-Linear Problem

We first specify the general non-linear problem in order to establish the conditions under which the linear-quadratic approximation is an accurate approximation.

Consider the following optimisation problem:

\[
Max \quad U(y) \quad (A.1)
\]

\[
s.t.: \quad G(y) = 0,
\]

where \( U \) is the non-linear objective function, \( G \) is the vector of \( m \) non-linear constraints, and \( y \) is the vector of variables where for convenience we consider controls and states jointly. A dynamic problem can be accommodated in this notation by appropriately defining \( U, G, \) and \( y \).\(^A.1\)

Taking first order conditions one obtains:

\[
U_y + \gamma'G_y = 0, \quad (A.2)
\]

where \( \gamma \) is a vector of Lagrange multipliers. After linearising the first order conditions and the constraints, one obtains:

\[
(y - \bar{y})' \bar{U}_{yy} + (\gamma - \bar{\gamma})' \bar{G}_{y} + \sum_m \bar{\gamma}_m (y - \bar{y})' \bar{G}_{yy}^m = 0, \quad (A.3)
\]

\[
(y - \bar{y})' \bar{G}_{y} = 0, \quad (A.4)
\]

where variables and functions with bars are evaluated at the steady state. The system of equations determines the solution of the non-linear system where the laws of motion are approximated to

\(^A.1\) Note that we suppress the time-dimension as a dynamic problem unnecessarily complicates the notation without materially changing the results. The only additional feature is that one needs to consider the timeless perspective (see Woodford, 2003) for the linear-quadratic approximation to be valid. Details are available upon request from the authors.
first order. In a dynamic context, standard techniques can be used to compute the solution to this system of equations, for instance the method outlined by Anderson and Moore (1985).

A.2 Linear-Quadratic Approximation: A General Approach

A second-order approximation to the utility function yields

\[ U(y) = U_y (y - \bar{y}) + \frac{1}{2} (y - \bar{y})' U_{yy} (y - \bar{y}), \tag{A.5} \]

and a second-order approximation to a constraint \( m \) yields

\[ G^m_y = G^m_y (y - \bar{y}) + \frac{1}{2} (y - \bar{y})' G^m_{yy} (y - \bar{y}) = 0. \tag{A.6} \]

One can sum equation (A.5) and equations (A.6) for each \( m \) with weights 1 and \( \bar{\gamma}' \). This operation is valid since the constraints are equal to zero. In this case we obtain:

\[ U(y) = U_y (y - \bar{y}) + \frac{1}{2} (y - \bar{y})' U_{yy} (y - \bar{y}) + \sum_m \bar{\gamma}_m \left[ G^m_y (y - \bar{y}) + \frac{1}{2} (y - \bar{y})' G^m_{yy} (y - \bar{y}) \right]. \tag{A.7} \]

Noting that \( U_y + \sum_m \bar{\gamma}_m G^m_y = U_y + \bar{\gamma}' G_y = 0 \) where the last equality comes from using equation (A.2) at the steady state, one can simplify equation (A.7) further:

\[ U(y) = \frac{1}{2} (y - \bar{y})' U_{yy} (y - \bar{y}) + \sum_m \bar{\gamma}_m \frac{1}{2} (y - \bar{y})' G^m_{yy} (y - \bar{y}). \tag{A.8} \]

Now use the transformed objective function (A.8) in the maximisation problem:

\[
\max_y \frac{1}{2} (y - \bar{y})' U_{yy} (y - \bar{y}) + \sum_m \bar{\gamma}_m \frac{1}{2} (y - \bar{y})' G^m_{yy} (y - \bar{y}) \tag{A.9}
\]

\[ s.t. : (y - \bar{y})' G_y = 0. \]

Taking first order conditions one obtains:

\[ (y - \bar{y})' U_{yy} + \gamma' G_y + \sum_m \bar{\gamma}_m (y - \bar{y})' G^m_{yy} = 0. \tag{A.10} \]

Since equation (A.10) is equal to equation (A.3), which is valid in any model, we can conclude that our approach is valid in general, even in models with a distorted steady state.\(^A.2\) This is the approach we employ in the paper since it also delivers correct results with a distorted steady state. The reader is referred to Benigno and Woodford (2012) for additional details.\(^A.2\) Note it is immaterial to write \( (\gamma - \bar{\gamma})' G_y \) instead of \( \gamma' G_y \).

\(^A.2\)
A.3 Linear-Quadratic Approximation: A Simple Approach for a Non-Distorted Steady State

There is a simpler approach but it only delivers correct results if the steady state is non-distorted. The problem of maximising the second-order approximation to utility in equation (A.5) subject to a first-order approximation to the constraints is

\[
\max_y \hat{U}_y (y - \bar{y}) + \frac{1}{2} (y - \bar{y})' \hat{U}_{yy} (y - \bar{y}) \tag{A.11}
\]

\[
\text{s.t. } (y - \bar{y})' \hat{G}_y = 0.
\]

Taking first order conditions one obtains:

\[
\hat{U}_y + (y - \bar{y})' \hat{U}_{yy} + \gamma' \hat{G}_y = 0. \tag{A.12}
\]

Equation (A.3) is directly comparable with equation (A.12). As is easily seen, this LQ approach does not usually give the correct solution.\(^{A.3}\) Benigno and Woodford (2012) referred to this alternative linear-quadratic approximation as a “naive” LQ approximation.

In special circumstances, the direct approach leading to equation (A.12) yields the correct solution. This is the case when the economy at the steady state is at the unconstrained optimum. Also one needs to use substitution of variables such that all market clearing conditions and feasibility are not present in \(G(y)\). That means that:

\[
\hat{U}_y = 0, \tag{A.13}
\]

and hence according to equation A.2:

\[
\gamma' \hat{G}_y = 0. \tag{A.14}
\]

In this case, equations (A.12) and (A.10) coincide, and are given by:

\[
(y - \bar{y})' \hat{U}_{yy} = 0. \tag{A.15}
\]

Note that it is not required that the economy is always at the unconstrained optimum. It suffices that is the case at the steady state.\(^{A.4}\) This approach is used for instance in Levine, McAdam and Pearlman (2008).

---

\(^{A.3}\) This incorrect result is the first of the two pitfalls of linearisation methods discussed in Kim and Kim (2007).

\(^{A.4}\) Note that we have abused notation by not having distinguished explicitly endogenous and exogenous variables. While this distinction is important, it would complicate the notation without changing the intuition. For details see Benigno and Woodford (2012). In our notation, we can always append the exogenous variables to the vector \(y\).
In our case, the difference between output with the distorted and non-distorted steady state is 6%. Woodford (2003) shows that if distortions are small then the optimal response to economic shocks does not change. Still, we employ the approach that can handle the distorted steady state since this is the empirically more realistic benchmark.

Appendix B Additional Analytical Results

B.1 FOCs in Canonical Sticky Price and Wage Model

Minimising the loss function (21), subject to (18)-(20) one obtains the first-order conditions

\[ \pi_t^p - \Delta \varsigma_{1,t} + \varsigma_{3,t} = 0 \]  
(B.1)

\[ \lambda_{w, \pi_t}^{opt} \lambda_{w, \pi_t}^{opt} = \Delta \varsigma_{2,t} - \varsigma_{3,t} = 0 \]  
(B.2)

\[ \lambda_{w, y_t}^{opt} y_t^{gap} + \kappa_p \varsigma_{1,t} + \kappa_w \varsigma_{2,t} = 0 \]  
(B.3)

\[ \vartheta_p \varsigma_{1,t} - \vartheta_w \varsigma_{2,t} + \varsigma_{3,t} - \beta E_t \varsigma_{3,t+1} = 0, \]  
(B.4)

where \( \varsigma_{1,t}, \varsigma_{2,t}, \varsigma_{3,t} \) are Lagrange multipliers. In the special case with \( \kappa_p = \kappa_w \equiv \kappa \) and \( \lambda_{w, w}^{opt} = \vartheta_p/\vartheta_w \), combining eqs. (B.1)-(B.3) gives the targeting rule

\[ \vartheta_w \pi_t^p + \vartheta_p \pi_t^w = -\frac{\lambda_{w, \pi_t}^{opt}}{\kappa} \vartheta_w \left( y_t^{gap} - y_{t-1}^{gap} \right), \]

which coincides with eq. (24) in the main text.

B.2 Inefficient Cost-Push Shocks

We discuss here the impact of cost-push shocks. In order to do this, we consider a simplified version of the model with perfect competition in the labour market. But the results we present below generalise to the case with sticky wages and exogenous wage markup shocks.

In the standard New Keynesian model with sticky prices only, the Ramsey policy is obtained by minimizing

\[ L_{Ramsey} = \min_{\{\pi_t, y_t^{gap}\}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \lambda_{w, \pi_t}^{opt} (y_t^{gap})^2 \right] \]

s.t. \( \pi_t = \beta E_t \pi_{t+1} + \kappa_y y_t^{gap} + u_t, \)

where \( u_t \) in the second equation represents a cost-push shock, i.e. inefficient exogenous variations in the markup of the firms in the monopolistic goods sector. The laws of motion of the economy
are given by the NK Phillips curve and the optimality condition

\[ \pi_t = -\frac{\lambda^{opt}}{\kappa} (y_t^{gap} - y_{t-1}^{gap}) \]  

(B.7)

for \( t \geq 1 \) and \( \pi_0 = -\frac{\lambda^{opt}}{\kappa} y_0^{gap} \) for \( t = 0 \). With cost-push shocks, the solution of the Ramsey system is:

\[ y_0^{gap} = -\frac{\kappa \delta}{\lambda^{opt} (1 - \delta \beta \rho_u)} u_0 \]  

(B.8)

\[ y_t^{gap} = \delta y_{t-1}^{gap} - \frac{\kappa \delta}{\lambda^{opt} (1 - \delta \beta \rho_u)} u_t, \]  

(B.9)

where \( \delta \equiv \frac{1-\sqrt{1-4\beta \rho_u^2}}{2\alpha \beta} \) and \( a \equiv \frac{\lambda}{\lambda^{opt} (1 + \beta) + \kappa^2} \). For a central bank with a mandate in which the weight on the output gap is \( \lambda \), the laws of motion take the same functional form but where \( \lambda^{opt} \) is substituted by \( \lambda \). Hence, it is easy to see that if the central bank’s \( \lambda \) differs from \( \lambda^{opt} \), the solution for the simple mandate does not mimic that under the optimal policy. An important implication is that complete stabilisation of the output gap is generally non-optimal when cost-push shocks are present.

In this model, the Blanchard-Galí’s (2008) divine coincidence result only holds when cost push shocks are not present, that is \( \sigma(u_t) = 0 \). Without cost-push shocks, the solution is given by \( \pi_t = y_t^{gap} = 0 \) for any weight \( \lambda \geq 0 \). Thus, the simple mandate mimics the optimal policy for any choice of \( \lambda \). The same result holds in a model with sticky-wages and flexible prices. This result clarifies why in Figure 3 without inefficient shocks (the blue dashed line) welfare as a function of \( \lambda \geq 0 \) is essentially flat and why there is curvature with the benchmark calibration.

**Appendix C  The Smets and Wouters (2007) Model**

Below, we describe the firms’ and households’ problem in the model, and state the market clearing conditions.\(^{C.1}\)

\(^{C.1}\) For a description of the model which derives the log-linearised equations, we refer the reader to the appendix of the Smets and Wouters paper, which is available online at http://www.aeaweb.org/aer/data/june07/20041254_app.pdf.
Following Dotsey and King (2005) and Levin, López-Salido and Yun (2007) we assume that \( G_Y(\cdot) \) is given by a strictly concave and increasing function; its particular parameterisation follows SW:

\[
G_Y\left(\frac{Y_t(f)}{Y_t}\right) = \left(\frac{\phi_p}{1-(\phi_p-1)\epsilon_p}\right)^{1-(\phi_p-1)\epsilon_p}\left(\frac{1-\epsilon_p}{\phi_p}\right)^{\phi_p^{-1}}+\left(\frac{1-\epsilon_p}{1-(\phi_p-1)\epsilon_p}\right)^{1-(\phi_p-1)\epsilon_p}+\left[1-\left(\frac{\phi_p}{1-(\phi_p-1)\epsilon_p}\right)^{\phi_p^{-1}}\right],
\]

(C.2)

where \( \phi_p \geq 1 \) denotes the gross markup of the intermediate firms. The parameter \( \epsilon_p \) governs the degree of curvature of the intermediate firm’s demand curve. When \( \epsilon_p = 0 \), the demand curve exhibits constant elasticity as with the standard Dixit-Stiglitz aggregator. When \( \epsilon_p \) is positive—as in SW—this introduces more strategic complementarity in price setting which causes intermediate firms to adjust prices less to a given change in marginal cost.

Firms that produce the final output good \( Y_t \) are perfectly competitive in both the product and factor markets, and take as given the price \( P_t(f) \) of each intermediate good \( Y_t(f) \). They sell units of the final output good at a price \( P_t \), and hence solve the following problem:

\[
\max_{\{Y_t, Y_t(f)\}} P_t Y_t - \int_0^1 P_t(f) Y_t(f) df,
\]

subject to the constraint (C.1).

**Intermediate Goods Production** A continuum of intermediate goods \( Y_t(f) \) for \( f \in [0, 1] \) is produced by monopolistically competitive firms, which utilise capital services \( K_t(f) \) and a labour index \( L_t(f) \) (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

\[
Y_t(f) = \varepsilon_t^a K_t(f)^\alpha [\gamma^t L_t(f)]^{1-\alpha} - \gamma^t \Phi,
\]

(C.4)

where \( \gamma^t \) represents the labour-augmenting deterministic growth rate in the economy, \( \Phi \) denotes the fixed cost (which is related to the gross markup \( \phi_p \) so that profits are zero in the steady state), and \( \varepsilon_t^a \) is total factor productivity which follows the process

\[
\ln \varepsilon_t^a = (1 - \rho_a) \ln \varepsilon^{a_0} + \rho_a \ln \varepsilon_{t-1}^a + \eta_t^a, \eta_t^a \sim N(0, \sigma_a).
\]

(C.5)

Firms face perfectly competitive factor markets for renting capital at price \( R_{K_t} \) and hiring labour at a price given by the aggregate wage index \( W_t \) (defined below). As firms can costlessly adjust either factor of production, the standard static first-order conditions for cost minimisation imply that all firms have identical marginal cost per unit of output.

The prices of the intermediate goods are determined by Calvo-Yun (1996) style staggered nominal contracts. The probability \( 1 - \xi_p \) that any firm \( f \) receives a signal to re-optimise its price
\( P_t(f) \) is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimise its price, it adjusts its price by a weighted combination of the lagged and steady-state rate of inflation, i.e., \( P_t(f) = (1 + \pi_{t-1})^{1-p} (1 + \pi)^{1-t_p} P_{t-1}(f) \) where \( 0 \leq t_p \leq 1 \) and \( \pi_{t-1} \) denotes net inflation in period \( t-1 \), and \( \pi \) the steady-state net inflation rate. A positive value of \( t_p \) introduces structural inertia into the inflation process. All told, this leads to the following optimisation problem for the intermediate firms

\[
\max_{\tilde{P}_t(f)} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \xi_j)^j \frac{\tilde{P}_{t+j} P_t}{\tilde{P}_t P_{t+j}} \left[ \tilde{P}_t(f) \left( \Pi_{s=1}^{j} (1 + \pi_{t+s-1})^{1-t_p} (1 + \pi)^{1-t_p} \right) - MC_{t+j} \right] Y_{t+j}(f), \quad (C.6)
\]

where \( \tilde{P}_t(f) \) is the newly set price. Notice that with our assumptions all firms that re-optimise their prices actually set the same price.

It would be ideal if the markup in \((C.2)\) can be made stochastic and the model can be written in a recursive form. However, such an expression is not available, and we instead directly introduce a shock \( \varepsilon^p \) in the first-order condition to the problem in \((C.6)\). And following SW, we assume the shock is given by an exogenous ARMA(1,1) process:

\[
\ln \varepsilon^p_t = (1 - \rho_p) \ln \varepsilon^p + \rho_p \ln \varepsilon^p_{t-1} + \eta^p_t - \mu_p \eta^p_{t-1}, \eta^p_t \sim N(0, \sigma_p). \quad (C.7)
\]

When this shock is introduced in the non-linear model, we put a scaling factor on it so that it enters exactly the same way in a log-linearised representation of the model as the price markup shock does in the SW model.\(^{C.2}\)

\( C.2 \) Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labour service to the production sector; that is, goods-producing firms regard each household’s labour services \( L_t(h) \), \( h \in [0, 1] \), as imperfect substitutes for the labour services of other households. It is convenient to assume that a representative labour aggregator combines households’ labour hours in the same proportions as firms would choose. Thus, the aggregator’s demand for each household’s labour is equal to the sum of firms’ demands. The aggregated labour index \( L_t \) has the Kimball (1995) form:

\[
L_t = \int_0^1 G_L \left( \frac{L_t(h)}{L_t} \right) dh = 1, \quad (C.8)
\]

\(^{C.2}\) Alternatively, we could have followed the specification in Adjemian et al. (2008) and introduced the shock as a tax on the intermediate firm’s revenues in the problem \((C.6)\) directly. The drawback with this alternative approach is that the log-linearised representation of the model would have a different lead-lag structure from the representation in SW. In Section 3, we perform robustness analysis with respect to the price- and wage-markup shocks and show that our main result holds.
where the function $G_L(\cdot)$ has the same functional form as (C.2), but is characterised by the corresponding parameters $\epsilon_w$ (governing convexity of labour demand by the aggregator) and $\phi_w$ (gross wage markup). The aggregator minimises the cost of producing a given amount of the aggregate labour index $L_t$, taking each household’s wage rate $W_t(h)$ as given, and then sells units of the labour index to the intermediate goods sector at unit cost $W_t$, which can naturally be interpreted as the aggregate wage rate.

The utility function of a typical member of household $h$ is

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1 - \sigma} (C_{t+j}(h) - \kappa C_{t+j-1}) \right]^{1-\sigma} \exp \left( \frac{\sigma_c - 1}{1 + \sigma} L_{t+j}(h)^{1+\sigma} \right),$$

(C.9)

where the discount factor $\beta$ satisfies $0 < \beta < 1$. The period utility function depends on household $h$’s current consumption $C_t(h)$, as well as lagged aggregate per capita consumption to allow for external habit persistence through the parameter $0 \leq \kappa \leq 1$. The period utility function also depends inversely on hours worked $L_t(h)$.

Household $h$’s budget constraint in period $t$ states that its expenditure on goods and net purchases of financial assets must equal its disposable income:

$$P_t C_t(h) + P_t I_t(h) + \frac{B_{t+1}(h)}{\epsilon^h_t R_t} + \int_s \xi_{t,t+1} B_{D,s,t+1}(h) - B_{D,t}(h)$$

$$= B_t(h) + W_t(h) L_t(h) + R_t^h Z_t(h) K^P_t(h) - a(Z_t(h)) K^P_t(h) + \Gamma_t(h) - T_t(h).$$

(C.10)

Thus, the household purchases part of the final output good (at a price of $P_t$), which it chooses either to consume $C_t(h)$ or invest $I_t(h)$ in physical capital. Following Christiano, Eichenbaum, and Evans (2005), investment augments the household’s (end-of-period) physical capital stock $K^P_{t+1}(h)$ according to

$$K^P_{t+1}(h) = (1 - \delta) K^P_t(h) + \epsilon^i_t \left[ 1 - S \left( \frac{I_t(h)}{I_{t-1}(h)} \right) \right] I_t(h).$$

(C.11)

The extent to which investment by each household $h$ turns into physical capital is assumed to depend on an exogenous shock $\epsilon^i_t$ and how rapidly the household changes its rate of investment according to the function $S \left( \frac{I_t(h)}{I_{t-1}(h)} \right)$, which we specify as

$$S(x_t) = \frac{\varphi}{2} (x_t - \gamma)^2.$$  

(C.12)

Notice that this function satisfies $S(\gamma) = 0$, $S'(\gamma) = 0$ and $S''(\gamma) = \varphi$. The stationary investment-specific shock $\epsilon^i_t$ follows

$$\ln \epsilon^i_t = \rho_t \ln \epsilon^i_{t-1} + \eta^i_t, \eta^i_t \sim N(0, \sigma_i).$$  

(C.13)
In addition to accumulating physical capital, households may augment their financial assets through increasing their government nominal bond holdings \((B_{t+1})\), from which they earn an interest rate of \(R_t\). The return on these bonds is also subject to a risk-shock, \(\xi_t\), which follows

\[
\ln \xi_t = \rho \ln \xi_{t-1} + \eta_t, \eta_t \sim N(0, \sigma) .
\]  

(C.14)

Agents can engage in frictionless trading of a complete set of contingent claims to diversify away idiosyncratic risk. The term \(R_s B_{D,t+1}(h)\) represents net purchases of these state-contingent domestic bonds, with \(\xi_{t,t+1}\) denoting the state-dependent price, and \(B_{D,t+1}(h)\) the quantity of such claims purchased at time \(t\).

On the income side, each member of household \(h\) earns after-tax labour income \(W_t(h)\), after-tax capital rental income of \(R^k Z_t(h) K^p_t(h)\), and pays a utilisation cost of the physical capital equal to \(a(Z_t(h)) K^p_t(h)\) where \(Z_t(h)\) is the capital utilisation rate, so that capital services provided by household \(h\), \(K_t(h)\), equals \(Z_t(h) K^p_t(h)\). The capital utilisation adjustment function \(a(Z_t(h))\) is assumed to be given by

\[
a(Z_t(h)) = \frac{r^k}{z_1} \left[ \exp \left( z_1 (Z_t(h) - 1) \right) - 1 \right],
\]  

(C.15)

where \(r^k\) is the steady-state real rental rate \((\bar{R}^k / P_t)\) of capital. Notice that the adjustment function satisfies \(a(1) = 0\), \(a'(1) = r^k\), and \(a''(1) = r^k z_1\). Following SW, we want to write \(a''(1) r^k z_1 = 0\), where \(\psi \in [0, 1]\) and a higher value of \(\psi\) implies a higher cost of changing the utilisation rate. Our parameterisation of the adjustment cost function then implies that we need to set \(\tilde{z}_1 = 1 / z_1\). Finally, each member also receives an aliquot share \(\Gamma_t(h)\) of the profits of all firms, and pays a lump-sum tax of \(T_t(h)\) (regarded as taxes net of any transfers).

In every period \(t\), each member of household \(h\) maximises the utility function (C.9) with respect to its consumption, investment, (end-of-period) physical capital stock, capital utilisation rate, bond holdings, and holdings of contingent claims, subject to its labour demand function, budget constraint (C.10), and transition equation for capital (C.11).

Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described previously. Thus, the probability that a household receives a signal to re-optimise its wage contract in a given period is denoted by \(1 - \xi_w\). In addition, SW specify the following dynamic indexation scheme for the adjustment of the wages of those households that do not get a signal to re-optimise: \(W_t(h) = \gamma (1 + \pi_{t-1})^{\xi_w} (1 + \pi)^{1 - \xi_w} W_{t-1}(h)\). All told, this leads
to the following optimisation problem for the households

$$
\max_{\tilde{W}(h)} E_t \sum_{j=0}^{\infty} (\beta^{\xi_w})^j \frac{\Xi_{t+j} P_t}{\Xi_{t} P_{t+j}} \left[ \tilde{W}_t(h) \left( \Pi_{s=1}^j (1 + \pi_{t+s-1})^{\zeta_w} (1 + \pi)^{1-\zeta_w} \right) - W_{t+j} \right] L_{t+j}(h),
$$

where $\tilde{W}_t(h)$ is the newly set wage; notice that with our assumptions all households that re-optimise their wages will actually set the same wage.

Following the same approach as with the intermediate-goods firms, we introduce a shock $\varepsilon_t^w$ in the resulting first-order condition. This shock, following SW, is assumed to be given by an exogenous ARMA(1,1) process

$$
\ln \varepsilon_t^w = (1 - \rho_w) \ln \varepsilon_t^w + \rho_w \ln \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w, \eta_t^w \sim N(0, \sigma_w).
$$

As discussed previously, we use a scaling factor for this shock so that it enters in exactly the same way as the wage markup shock in SW in the log-linearised representation of the model.

C.3 Market Clearing Conditions

Government purchases $G_t$ are exogenous, and the process for government spending relative to trend output, i.e. $g_t = G_t / (\gamma Y)$, is given by the following exogenous AR(1) process:

$$
\ln g_t = (1 - \rho_g) \ln g + \rho_g \left( \ln g_{t-1} - \rho_{ga} \ln \varepsilon_t^a \right) + \varepsilon_t^g, \varepsilon_t^g \sim N(0, \sigma_g).
$$

Government purchases have no effect on the marginal utility of private consumption, nor do they serve as an input into goods production. Moreover, the government is assumed to balance its budget through lump-sum taxes (which are irrelevant since Ricardian equivalence holds in the model).

Total output of the final goods sector is used as follows:

$$
Y_t = C_t + I_t + G_t + a(Z_t) \bar{K}_t,
$$

where $a(Z_t) \bar{K}_t$ is the capital utilisation adjustment cost.

Finally, one can derive an aggregate production constraint, which depends on aggregate technology, capital, labour, fixed costs, as well as the price and wage dispersion terms.$^{C.3}$

C.4 Model Parameterisation

When solving the model, we adopt the parameter estimates (posterior mode) in Tables 1.A and 1.B of SW. We also use the same values for the calibrated parameters. Table A1 provides the relevant values.

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$^{C.3}$ We refer the interested reader to Adjemian, Paries and Moyen (2008) for further details.
Table C.1: Parameter Values in Smets and Wouters (2007).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<th>Description</th>
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<td>10</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>Gross wage markup</td>
<td>1.50</td>
<td>$\epsilon_w$</td>
<td>Kimball Elast. LM</td>
<td>10</td>
</tr>
<tr>
<td>$g_y$</td>
<td>Gov’t $G/Y$ ss-ratio</td>
<td>0.18</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Estimated

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>Investment adj. cost</td>
<td>5.48</td>
<td>$\alpha$</td>
<td>Capital production share</td>
<td>0.19</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Inv subs. elast. of cons.</td>
<td>1.39</td>
<td>$\psi$</td>
<td>Capital utilisation cost</td>
<td>0.54</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Degree of ext. habit</td>
<td>0.71</td>
<td>$\phi_p$</td>
<td>Gross price markup</td>
<td>1.61</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Calvo prob. wages</td>
<td>0.73</td>
<td>$\pi$</td>
<td>Steady-state net infl. rate</td>
<td>0.0081</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>Labour supply elas.</td>
<td>1.92</td>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9984</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>Calvo prob. prices</td>
<td>0.65</td>
<td>$l$</td>
<td>Steady-state hours worked</td>
<td>0.25</td>
</tr>
<tr>
<td>$\nu_w$</td>
<td>Ind. for non-opt. wages</td>
<td>0.59</td>
<td>$\gamma$</td>
<td>Steady-state gross growth</td>
<td>1.0043</td>
</tr>
<tr>
<td>$\nu_p$</td>
<td>Ind. for non-opt. prices</td>
<td>0.22</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel C: Shock Processes

<table>
<thead>
<tr>
<th>Shock</th>
<th>Persistence</th>
<th>MA(1)</th>
<th>Std. of Innovation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutral Technology</td>
<td>$\rho_a$</td>
<td>0.95</td>
<td>$\sigma_a$</td>
</tr>
<tr>
<td>Risk premium</td>
<td>$\rho_b$</td>
<td>0.18</td>
<td>$\sigma_b$</td>
</tr>
<tr>
<td>Gov’t spending</td>
<td>$\rho_g$</td>
<td>0.97</td>
<td>$\rho_{ga}$</td>
</tr>
<tr>
<td>Inv. Specific Tech.</td>
<td>$\rho_i$</td>
<td>0.71</td>
<td>$\sigma_i$</td>
</tr>
<tr>
<td>Price markup</td>
<td>$\rho_p$</td>
<td>0.90</td>
<td>$\mu_p$</td>
</tr>
<tr>
<td>Wage markup</td>
<td>$\rho_w$</td>
<td>0.97</td>
<td>$\mu_w$</td>
</tr>
<tr>
<td>Monetary policy</td>
<td>$\rho_r$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes. SW estimates $\rho_r = 0.12$ and $\sigma_r = 0.24$, but in our optimal policy exercises these parameters are not present.

There are two issues to notice with regards to the parameters in Table C1. First, we adapt and re-scale the processes of the price and wage markup shocks so that when our model is log-linearised it matches exactly the original SW model. Second, we set the monetary policy shock parameters to nil, as we restrict our analysis to optimal policy.

Appendix D  Role of Labour Market Variables

Recent influential theoretical papers support that literature by suggesting to add wage inflation as an additional target variable in the loss function, see e.g. Galí (2011) and our previous analysis of the EHL model in Section 2.1. In the SW model employed in our analysis, both nominal wages and prices are sticky. It is therefore conceivable that wage inflation may be equally or even more important to stabilise than price inflation. In addition to studying nominal wage inflation, it is of interest to examine to what extent other labour market variables like employment or hours worked
can substitute for overall economic activity within the model. Hence, we propose to study the following augmented loss function:

\[ L_t^a = \lambda_\pi^a (\pi_t^a - \pi^a)^2 + \lambda_\Delta^a (\Delta \pi_t^a - \Delta \pi^a)^2 + \lambda_\delta^a \Delta w_t^a, \quad (D.1) \]

where \( \Delta w_t^a \) denotes annualised nominal wage inflation (and \( \Delta w^a \) its steady-state rate of growth), and \( e_t \) involves a measure of activity in the labour market.

In results not shown, we have found that the optimised weights in the tri-variate loss function in the second row of Table D.1 change little with respect to the inefficient markup shocks. Given that the weights in a simple mandate have unique optimal weights only when it mimics Ramsey policy (see Section 3), this finding suggests that the tri-variate loss function approximates Ramsey policy very closely. Accordingly, this loss function—which features a high weight on the output gap—supports the finding in Table 2 that the share of efficient and inefficient shocks does not change the overall message that the weight on the output-gap should be high.

In our framework with inefficient cost-push shocks and capital accumulation, the introduction of \( \Delta w_t^a \) in the loss function does not make the presence of \( y_t^{gap} \) irrelevant, supporting the results we established in Section 2.1 with the EHL model. The third row makes this clear by showing that the

\[ L_t^a = \lambda_\pi^a (\pi_t^a - \pi^a)^2 + \lambda_\Delta w_t^a (\Delta w_t^a - \Delta w^a)^2 + \lambda_\delta^a \Delta w_t^a, \quad (D.1) \]

where \( \lambda_\delta^a \) denotes annualised nominal wage inflation (and \( \Delta w^a \) its steady-state rate of growth), and \( e_t \) involves a measure of activity in the labour market.

Table D.1: Variations of the Loss Function: Gap Variables in (D.1).

<table>
<thead>
<tr>
<th>Loss Function</th>
<th>( \lambda_\pi^a )</th>
<th>( \lambda_\Delta w_t^a )</th>
<th>( \lambda_\delta^a )</th>
<th>CEV (%)</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>1.000</td>
<td>1.042</td>
<td>-</td>
<td>0.044</td>
<td>-</td>
</tr>
<tr>
<td>Adding ( \Delta w_t^a )</td>
<td>1.000</td>
<td>3.216</td>
<td>1.485</td>
<td>0.029</td>
<td>32.8%</td>
</tr>
<tr>
<td>Adding ( \Delta w_t^a ), impose ( \lambda^a = 0.01 )</td>
<td>1.000</td>
<td>0.01*</td>
<td>0.013</td>
<td>1.260</td>
<td>-2673.6%</td>
</tr>
<tr>
<td>Replacing ( \pi_t ) with ( \Delta w_t^a )</td>
<td>-</td>
<td>1.546</td>
<td>1.000</td>
<td>0.032</td>
<td>27.3%</td>
</tr>
<tr>
<td>Adding ( y_t^{gap} )</td>
<td>1.000</td>
<td>0.880</td>
<td>-</td>
<td>0.518</td>
<td>0.043</td>
</tr>
<tr>
<td>Replacing ( y_t^{gap} ) with ( l_t^{gap} )</td>
<td>1.000</td>
<td>-</td>
<td>3.250</td>
<td>0.050</td>
<td>-14.3%</td>
</tr>
<tr>
<td>Replacing ( \pi_t, y_t^{gap} ) with ( [\Delta w_t^a, l_t^{gap}] )</td>
<td>-</td>
<td>-</td>
<td>1.000</td>
<td>0.016</td>
<td>63.3%</td>
</tr>
</tbody>
</table>

Notes. The table reports variations of the simple objective (D.1). \( y_t^{gap} \) is used as the measure of \( x_t \), and \( l_t^{gap} \) is used as the measure of \( e_t \). The numbers in the “Gain” column are computed as 100 \( \left( 1 - \frac{\text{CEV}_{LFalt}}{\text{CEV}_{LFalt}} \right) \), where CEV_{LFalt} is the CEV for the alternative loss function and 0.044 is the “Benchmark” objective CEV (row 1). A "*" after a coefficient implies that the value of this coefficient has been imposed.

In Table D.1, we report results for this augmented loss function (D.1) when \( x_t \) is given by the output gap and \( e_t \) is given by the hours worked per capita gap \( l_t^{gap} \), respectively. The labour market gap, defined as \( l_t^{gap} = l_t - l_t^{pot} \), differs from the output gap because of the presence of capital in the production function. The first row re-states the benchmark results, i.e. with the optimised weight on \( y_t^{gap} \) in Table 1. The second row adds wage inflation to the loss function. Relative to the unit weight on inflation, the optimised objective function would ask for a weight of roughly 3.2 for the output gap term, and a weight of about 1.5 for nominal wage inflation volatility, which is higher than the normalised weight on price inflation volatility. In line with Levin et al. (2005), the level of welfare when adding \( \Delta w_t \) is substantially higher (by 32.8 percent, when measured by the decrease in loss) than under the benchmark case.\(^{D.1}\)

\(^{D.1}\)In results not shown, we have found that the optimised weights in the tri-variate loss function in the second row of Table D.1 change little with respect to the inefficient markup shocks. Given that the weights in a simple mandate have unique optimal weights only when it mimics Ramsey policy (see Section 3), this finding suggests that the tri-variate loss function approximates Ramsey policy very closely. Accordingly, this loss function—which features a high weight on the output gap—supports the finding in Table 2 that the share of efficient and inefficient shocks does not change the overall message that the weight on the output-gap should be high.
welfare loss is very high for a mandate which includes both price and wage inflation but imposes a low weight on the output gap.\textsuperscript{D.2} Moreover, we learn from the fourth row in the table that, although $\Delta w_t^o$ receives a larger coefficient than $\pi_t^o$, responding to price inflation is still welfare enhancing; when dropping $\pi_t^o$ the welfare gain is somewhat lower than in the trivariate loss function. Also, the optimal weight on economic activity remains high.

Figure D.1: CEV (in percentage points) as Function of $\lambda^a$ for Alternative Simple Mandates.

Notes. The figure plots the CEV (in %) for the simple mandate with price inflation and output gap (solid line) and wage inflation and labour gap (dashed line). The coordinate with an 'o' mark shows the CEV for the optimised weight.

The fifth column of Table D.1 adds the labour market gap as an additional target variable. Unlike wage inflation, the inclusion of the labour market gap by itself does not increase welfare much. Moreover, given that price inflation is the nominal anchor, replacing the output gap with the labour gap results in a welfare deterioration of about 14 percent relative to our benchmark specification as can be seen from the sixth row. However, when price inflation is also replaced by wage inflation as a target variable, the labour gap performs much better and generates a substantial welfare gain of 63 percent relative to our benchmark specification.

In Figure D.1, we plot CEV as a function of $\lambda^a$ for a simple mandate targeting price inflation and the output gap as well as a mandate targeting wage inflation and the labour market gap.

\textsuperscript{D.2} Notice that the results in footnote 19, which states that a loss function with price- and wage-inflation only is isomorphic to a loss function with the output gap, is contingent on equal $\kappa$'s and no inefficient shocks and hence do not apply here. Moreover, because the obtained weight for nominal wage inflation is close to nil when $\lambda^a$ is fixed to 0.01, the CEV reported in Table D.1 is about the same as the CEV reported for the lowest value of $\lambda^a$ in Figure 3 for the benchmark calibration, recalling that the figure shows CEVs for the output gap when varying $\lambda^a$ between 0.01 and 5. Accordingly, it follows from the discussion of the results in Figure 3 that a loss function with only price and wage inflation is not observationally equivalent to a loss function in which the output gap is included even when the inefficient markup shocks are excluded; the optimised weight on the output gap is large even in this case, consistent with the analysis in Section 2.1. However, the absolute difference in CEV is much smaller in these cases: CEV is 0.03 for the pure price-wage inflation mandate, whereas it equals 0.016 (cf. last row in Table 2) when the output gap is included with a large weight.
Interestingly, we see from the figure that $\lambda^a$ has to exceed 2 in order for the wage-labour simple mandate to dominate. So although the wage-labour gap mandate dominates the inflation-output gap mandate, the figure makes clear that a rather large $\lambda^a$ is required for this to happen; strict nominal wage inflation targeting is thus very costly for society in terms of welfare. On the other hand, a beneficial aspect of the wage inflation-labour gap mandate is that if $\lambda^a$ indeed exceeds this threshold, then the CEV stays essentially flat instead of slightly increasing as is the case for the inflation-output gap mandate.

### Table D.2: Variations of the Loss Function: Level Variables in (D.1).

<table>
<thead>
<tr>
<th>Loss Function</th>
<th>$\lambda^a$: $\pi_t^a$</th>
<th>$\lambda^a$: $y_t - \bar{y}_t$</th>
<th>$\lambda^a_{\Delta w_t}$</th>
<th>$\lambda^a_{l_t - \bar{l}}$</th>
<th>CEV (%)</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>1.000</td>
<td>0.544</td>
<td>-</td>
<td>-</td>
<td>0.244</td>
<td>-</td>
</tr>
<tr>
<td>Adding $\Delta w_t^a$</td>
<td>1.000</td>
<td>0.954</td>
<td>0.463</td>
<td>-</td>
<td>0.230</td>
<td>5.3%</td>
</tr>
<tr>
<td>Replacing $\pi_t$ with $\Delta w_t^a$</td>
<td>-</td>
<td>1.054</td>
<td>1.000</td>
<td>-</td>
<td>0.246</td>
<td>-1.0%</td>
</tr>
<tr>
<td>Adding $l_t - \bar{l}$</td>
<td>1.000</td>
<td>0.392</td>
<td>-</td>
<td>1.344</td>
<td>0.171</td>
<td>29.8%</td>
</tr>
<tr>
<td>Replacing $y_t - \bar{y}_t$ with $l_t - \bar{l}$</td>
<td>1.000</td>
<td>-</td>
<td>-</td>
<td>2.947</td>
<td>0.210</td>
<td>13.8%</td>
</tr>
<tr>
<td>Replacing $[\pi_t, y_t - \bar{y}_t]$ with $[\Delta w_t^a, l_t - \bar{l}]$</td>
<td>-</td>
<td>-</td>
<td>1.000</td>
<td>3.475</td>
<td>0.161</td>
<td>33.8%</td>
</tr>
</tbody>
</table>

**Notes.** The table reports variations of the simple objective (D.1). $y_t - \bar{y}_t$ is used as the measure for $x_t$, and $l_t - \bar{l}$ is used as the measure of $e_t$. The numbers in the “Gain” column are computed as $100 \left( 1 - \frac{CEV_{LFalt}}{CEV_{Bench}} \right)$, where $CEV_{LFalt}$ is the CEV for the alternative loss function and 0.2440 is the “Benchmark” objective CEV (row 1).

We also examine the role of labour market variables when only observable variables are included; hence, we consider levels instead of gap variables. As shown in Table D.2, the role played by nominal wage inflation is not as prominent when $x_t$ in (D.1) is represented by the level of output (as deviation from a linear trend) instead of the output gap. The welfare gain relative to the benchmark case is only 5.3 percent higher when wage inflation is included. Accordingly, welfare is reduced by one percent—the third row—when price inflation is omitted. On the other hand, adding hours worked per capita enhances the welfare of households by nearly 30 percent. Finally, we see from the last row that a mandate with only wage inflation and hours worked performs the best, reducing the welfare cost associated with the simple mandate by nearly 34 percent relative to the benchmark objective.

**Appendix E Speed Limit Policies & Price- and Wage-Level Targeting**

In this appendix, we examine the performance of speed limit policies (SLP henceforth) advocated by Walsh (2003) and price- and wage-level targeting.

We start with an analysis of SLP. Walsh’s formulation of SLP considered actual growth relative to potential (i.e. output gap growth), but we also report results for actual growth relative to its
steady state to understand how contingent the results are on measuring the change in potential accurately. Moreover, since the results in the previous subsection suggested that simple mandates based on the labour market performed very well, we also study the performance of SLP for a labour market based simple mandate.

We report results for two parameterisations of the SLP objective in Table E.1. In the first row, we use the benchmark weight derived in Woodford (2003). In the second row, we adopt a weight that is optimised to maximise household welfare. Interestingly, we see that when replacing the level of output growth with the growth rate of the output gap ($\Delta y_t^{gap}$), welfare is increased substantially, conditional on placing a sufficiently large coefficient on this variable. However, by comparing these results with those for $y_t^{gap}$ in Table 1, we find it is still better to target the level of the output gap.

Turning to the SLP objectives based on nominal wage inflation and hours, we see that they perform worse than the standard inflation-output objectives unless the weight on the labour gap is sufficiently large. As is the case for output, the growth rate of the labour gap is preferable to the growth rate of labour itself. But by comparing these results with our findings in Table D.1 we see that targeting the level of the labour gap is still highly preferable in terms of maximising welfare of the households.

<table>
<thead>
<tr>
<th>Parameterisation</th>
<th>Price Inflation Objective</th>
<th></th>
<th>Wage Inflation Objective</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_t$: $\Delta y_t$</td>
<td>$x_t$: $\Delta y_t^{gap}$</td>
<td>$x_t$: $\Delta l_t$</td>
<td>$x_t$: $\Delta l_t^{gap}$</td>
</tr>
<tr>
<td>Woodford</td>
<td>0.048 CEV (%)</td>
<td>0.525 CEV (%)</td>
<td>0.048 CEV (%)</td>
<td>0.817 CEV (%)</td>
</tr>
<tr>
<td>Optimised</td>
<td>2.943 0.302</td>
<td>11.21 0.079</td>
<td>18.60 0.212</td>
<td>21.68 0.058</td>
</tr>
</tbody>
</table>

Notes. The loss function under price inflation is specified as in (30), while the loss function with the annualised nominal wage inflation rate wage is specified as $(\Delta w^a_t - \Delta w^a)^2 + \lambda^a x_t^2$, where $\Delta w^a$ denotes the annualised steady-state wage inflation rate; see eq. (D.1). $\Delta y_t$ denotes annualised output growth as deviation from the steady-state annualised growth rate ($4(\gamma - 1)$). $\Delta y_t^{gap}$ is the annualised rate of growth of output as deviation from potential, i.e. $4(\Delta y_t - \Delta y_t^{pot})$. The same definitions apply to hours worked. See the notes to Table 1 for further explanations.

Several important papers in the previous literature have stressed the merits of price level targeting as opposed to the standard inflation targeting loss function, see e.g. Vestin (2006). Price level targeting is a commitment to eventually bring back the price level to a baseline path in the face of shocks that create a trade-off between stabilising inflation and economic activity. Our benchmark flexible inflation targeting objective in eq. (30) can be replaced with a price level targeting objective as follows:

$$L^a_t = (p_t - \bar{p}_t)^2 + \lambda^a x_t^2,$$

where $p_t$ is the actual log-price level in the economy and $\bar{p}_t$ is the target log-price level path which
grows with the steady-state net inflation rate \( \pi \) according to \( \bar{p}_t = \pi + \bar{p}_{t-1} \). When we consider wage level targeting we adopt a specification isomorphic to that in (E.1), but replace the first term with \( w_t - \bar{w}_t \) where \( w_t \) is the nominal actual log-wage and \( \bar{w}_t \) is the nominal target log-wage which grows according to \( \bar{w}_t = \ln (\gamma) + \pi + \bar{w}_{t-1} \), where \( \gamma \) is the gross technology growth rate of the economy (see Table A.1).

In Table E.2, we report results for both price- and wage-level targeting objectives. As can be seen from the table, there are no welfare gains from pursuing price-level targeting relative to our benchmark objective in Table 2, regardless of whether one targets the output or the hours gap. For wage-level targeting, we obtain the same finding (in this case, the relevant comparison is the wage-inflation hours-gap specification in Table D.1 which yields a CEV of 0.016). These findings are perhaps unsurprising, given that the welfare costs in our model are more associated with changes in prices and wages (because of indexation) than with accumulated price- and wage-inflation rates.

### Table E.2: Sensitivity Analysis: Merits of Price and Wage Level Targeting.

<table>
<thead>
<tr>
<th>Parameterisation</th>
<th>Price-Level Targeting</th>
<th>Wage-Level Targeting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x_t: y_{tgap} )</td>
<td>( x_t: y_{tgap} )</td>
</tr>
<tr>
<td></td>
<td>( \lambda^g )</td>
<td>( \lambda^g )</td>
</tr>
<tr>
<td></td>
<td>CEV (%)</td>
<td>CEV (%)</td>
</tr>
<tr>
<td>Woodford</td>
<td>0.048 0.542 0.048 0.776</td>
<td>0.048 0.502 0.048 0.688</td>
</tr>
<tr>
<td>Optimised</td>
<td>9.187 0.092 28.41 0.095</td>
<td>11.37 0.064 36.14 0.067</td>
</tr>
</tbody>
</table>

**Notes.** The loss function under price-level targeting is given by (E.1), while the loss function with the nominal wage level is specified as \( L_t = (w_t - \bar{w}_t)^2 + \lambda^x x_t^2 \). See the notes to Table 1 for further explanations.

### Appendix F Robustness to Measurement Errors

In this appendix, we describe in more detail the analysis to examine if our results are robust to significant measurement errors of the output gap. Following Orphanides and Williams (2002), the true and observed potential levels of output evolve according to

\[
y_{t, pot} - y_{t, pot} = \rho \left( y_{t-1, pot} - y_{t-1} \right) + \varepsilon_t,
\]

with

\[
y_{t, pot} = FP_t \left( \{ y_j \}_{j=0}^{t} \right),
\]

\[
y_{t, pot, obs} = FP_t \left( \{ y_j \}_{j=0}^{T} \right),
\]

where \( FP_t \) denotes the element corresponding to time \( t \) of a filtering procedure \( FP \), and \( T \) denotes the last point of the sample. Equation (F.2) determines \( y_{t, pot, obs} \) and only makes use of data up to
time \( t \), which corresponds to real-time filtering (i.e. one-sided). Equation (F.3) determines true potential output \( y_{t}^{pot} \) by making use of all available data (i.e. two-sided).

By making use of the definitions

\[
y_{t}^{\text{gap,obs}} = y_{t} - y_{t}^{\text{pot,obs}}, \tag{F.4}
\]
\[
y_{t}^{\text{gap}} = y_{t} - y_{t}^{\text{pot}}, \tag{F.5}
\]

and by simple algebraic transformations on equation (F.1), we obtain equation (37) in the text. This equation can be estimated given eqs. (F.4) and (F.5), and estimating equation (37) or (F.1) is equivalent.

The procedure just described can readily be extended to account for data revisions, i.e. the fact that GDP is revised over time. To do that, we let \( y_{t|i} \) denote actual output in period \( t \) from vintage \( i \geq t \). The latest vintage of data is denoted by \( y_{t|T} \). We now employ the following definitions:

\[
y_{t|i}^{\text{pot,obs}} = FP_{i} \left( \{ y_{j|i} \}_{j=0}^{t} \right), \tag{F.6}
\]
\[
y_{t|T}^{\text{pot}} = FP_{T} \left( \{ y_{j|T} \}_{j=0}^{T} \right). \tag{F.7}
\]

Measurement error in potential output \( \left( y_{t|i}^{\text{pot,obs}} - y_{t|i|T}^{\text{pot}} \right) \) is now due to the availability of data (i.e. one-sided vs. two-sided filtering) plus the employment of real-time data with the associated revisions since in general \( y_{j|i} \neq y_{j|i|T} \). Besides the filtering problem in measuring potential output, data revisions also affect the output gap measurement directly since now eqs. (F.4) and (F.5) need to be modified accordingly to

\[
y_{t}^{\text{gap,obs}} = y_{t|i} - y_{t|i}^{\text{pot,obs}}, \tag{F.8}
\]
\[
y_{t}^{\text{gap}} = y_{t|T} - y_{t|T}^{\text{pot}}. \tag{F.9}
\]

One can estimate equation (37) directly given the definitions in eqs. (F.8) and (F.9). By accounting for both real-time filtering and data revisions, our approach differs from Orphanides and Williams (2002) who do not allow data revisions of actual GDP and how these revisions compound with the filtering problem itself.

As discussed in the main text, these procedures provide a benchmark for measurement error. However, the filtering step in equation (F.2)—for instance by an HP filter—does not necessarily deliver a measure of potential output that is consistent with the welfare- and model-consistent concept. This approach also does not incorporate model uncertainty and model misspecification. Still, this approach may be defensible provided that our estimate of measurement error is higher.
than the ones considered in the literature (and thus constitutes a tougher test for our key finding),
as well as for the arguments fleshed out in footnotes 37 and 38 in the main text.